International Journal of Engineering, Science and Mathematics
Vol. 9 Issue 2, February 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

OBTAINING THE VARIANCE AND THE COVARIANCES OF THE PARAMETER ESTIMATES OF THE SECOND ORDER EXPERIMENTAL DESIGN

Authors
KIPLAGAT JERONO MAGDALENE
University of Eldoret, Kenya

ABSTRACT

Rotatability is a property that requires that the variance of estimates of responses at points equidistant from the centre of the design is constant on circles or spheres or hyper-spheres. The study of rotatable designs mainly emphasises the estimation of the absolute response. In this study, the aim was to construct second order rotatable designs in three, four, five, six and k-factors based on balanced incomplete block designs. The main objective was to obtain the variance and covariances of parameter estimates of the second-order experimental designs. Using a balanced incomplete block design in three, four, five, six, and k-factors where each factor will contain two treatments, factorial combinations were obtained. An incidence matrix of Balanced Incomplete Block Design is suitably chosen and must satisfy the necessary Balanced Incomplete Block Design conditions. It should also satisfy the non-singularity conditions for a second-order design to be rotatable. A set of points $s(a,a,a)$ was also suitably chosen and used to denote the symmetric point sets associated with an appropriate, balanced incomplete design. In conclusion, some new second-order rotatable designs in three, four, five and six factors and their generalisation in K factors were obtained through balanced incomplete block designs.

INTRODUCTION

Response surface methodology (RS M) is a collection of statistical and mathematical techniques useful for developing, improving and optimising processes. The goal of most response surface research is to find a suitable approximating function for the purpose of predicting future response and to find levels of the input variables for which in some sense the response is optimised. The aim is to actually determine optimum operating conditions or to define a region in the space of the input variables where certain operating specifications are met.

A rotatable design is a series of response surface designs with the property that the variance of estimates of response at points equidistant from the centre of the design is constant. These designs ensure equal precision on the response estimates. The study of rotatable designs mainly emphasised on the estimation of absolute response.

Rotatability of designs has been studied by a number of researchers, Box and Hunter (1957) introduced rotatable designs for the exploration of response surfaces. He emphasised the estimation of absolute response, Victorbabu and Vasundharadevi (2009) studied on the efficiencies of second-order response surface designs for the estimation of response and slopes using symmetrical unequal block arrangement two unequal block sizes, Victorbabu (2006) constructed rotatable design using a pair of incomplete block design, Huda (1987) constructed third-order rotatable design from design of lower dimension. Some new third-order rotatable design in five dimensions through balanced incomplete block (BIB) designs has also been suggested by Koske, Mutiso and Kosgei (2011). In this study, some new second-order rotatable designs are obtained through a balanced incomplete block (BIB) designs. The method of construction will share some features proposed by Koske et al. (2011). Here the experimenter will start with three factors then four, five and six factors and later give a generalisation using k factors.

Balanced incomplete block (BIB) designs

According to Victorbabu (2006) and Victorbabu and Surekha (2011), a balanced incomplete block (BIB) design denoted by (v, b, r, s, λ) is an arrangement of v-treatments in b-blocks each containing s(< v) treatments and satisfying the following conditions.

a) Every treatment happens at most once in a block
b) Every treatment occurs in exactly r-blocks
c) Every pair of treatments occurs together in λ blocks

The quantities v, b, r, s and λ are called the parameters of a BIB design

The conditions necessary for the existence of a BIB design are;-

(i) $rv = sb$

(ii) $(r-1) = r(s-1)$

(iii) $r > \lambda$

(iv) $b \geq v$

Second-Order Rotatable Designs

Suppose we want to use a second response surface $D = x_{iu}$ to fit the surface

$$Y_u = b_o + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}{}^2 + \sum_{i \prec j} \sum b_{ij} x_{iu} x_{ju} + \ell_u$$

Where x_{iu} denotes the level of the i^{th} factor ($i = 1,2,......v$ in the u^{th} run ($u = 1,2,......N$ of the experiment e_u s are uncorrelated random errors with mean zeros and variance σ^2

b_0, b_i, b_{ii}, b_{ij} are the parameters of the model and y_u is the response observed at the u^{th} design point.

Then the surface is said to be a second-order rotatable arrangement if it satisfies the following two-moment conditions.

(i) $$\sum_{u=1}^{N} x_{iu}{}^2 = N\lambda_2 = A$$

(ii) $$\sum_{u=1}^{N} x_{iu}{}^4 = 3\sum_{u=1}^{N} x_{iu}{}^2 x_{ju}{}^2 = 3N\lambda_4 = 3B$$

Where λ_2 and λ_4 are constants, $A = \frac{N}{K}$, $B = \frac{N}{K(K+2)}$

A condition necessary for the existence of a non-singular second-order design is

$$\frac{\lambda_4}{\lambda_2{}^2} > \frac{K}{K+2} \text{ i.e. } \frac{NB}{A^2} > \frac{K}{K+2}$$

The necessary and sufficient conditions for a design to be a second-order rotatable design are required. The problem is to construct a generalised second-order rotatable design through balanced incomplete block design (BIBD) and to also obtain the variances and the covariances of the parameters estimates.

The study would be desirable where the experimenter is interested in some $k-1$ subset of factors that would give rise to a BIB ($k, b, s, k-1, \lambda$) and a replicate of the incidence matrix, where the second-order rotatable design in k dimension will have $k-1$a dimension that involves the subset of factors desired by the experimenter.

LITERATURE REVIEW

Response surface methodology (RS M) is a statistical technique very useful in the design and analysis of experiments. It involves a dependent variable y_u such as yield and is called the response variable.

In general, $y_u = f(x_{1u}, x_{2u}, \ldots\ldots x_{ku}) + e_u$ where $u = 1,2,\cdots\cdots.N$ represents the N-observations and x_{iu} is the level of the i^{th} factor in the u^{th} observation and y_u is the response, e_u is the random error with mean zero and variance σ^2. Response surface method is useful where several independent variables influence a dependent variable.

The concept of rotatability, which is very important in response surface second-order designs, was proposed by Box and Hunter (1957). The study emphasises on the estimation of absolute response. The k-dimensional point set forms a second-order rotatable arrangement in k- factors if the following conditions hold

$$\sum_{u=1}^{N} x_{iu}^{\ 2} = A \qquad i = (1,2\cdots\cdots k\ )$$

$$\sum_{u=1}^{N} x_{iu}^{\ 4} = 3\sum_{u=1}^{N} x_{iu}^{\ 2} x_{ju}^{\ 2} = 3B \tag{1}$$

and all other sums of powers and products up to order four are zero i.e. $\sum_{u=1}^{N}\prod_{i=1}^{v} x_{iu}\alpha_u = 0$

if any α is odd for $\sum \alpha_i \le 4$ and $A = N\lambda_2$, $B = N\lambda_4$

This arrangement of the points forms a non-singular second order rotatable designs if it satisfies the necessary condition of a second-order rotatable non-singular design, i.e. $\frac{NB}{A} \succ \frac{k}{k+2}$ which is the condition required for a second-order arrangement of points to form a second-order rotatable design, where also

$$A = \frac{N}{K} \text{ and } B = \frac{N}{K(K+2)} \tag{2}$$

According to Victorbabu and Vasundharaderi (2009), a design for fitting a response surface consists of a number of suitable combinations of levels of several input factors. He considered v factors and N combinations in the design, each having a constant number of levels. A response surface design can be written as v - rows and N columns each. Each row being a combination of v - levels codes on from each of v - ordered factors. This combination of level codes he called a design point, and the combination with 0- code for each factor is called a central point. They were considering the efficiencies of various second-order response surface designs. Victorbabu (2005) studied modified slope rotatable central composite designs (SRCCD). Victorbabu (2006) studied second order rotatable designs using a pair of incomplete block designs. He considered a second order rotatable design by taking combinations with unknown constants and associated a 2^k factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant to form a design. Victorbabu and Narasimham (1991) constructed second-order slope rotatable designs (SOSRD) using a balanced incomplete block design. Victorbabu (2009) also reviewed the modified SOSRDs. He also presented different methods of construction of modified SOSRDs using central composite designs, Balanced incomplete block design, pairwise balanced designs symmetrical unequal block arrangement with two unequal block sizes. Victorbabu and Surekha (2011) constructed a SOSRD using a balanced incomplete block design. The study of rotatable designs is mainly emphasised on the estimation of differences of yields and its precision. Estimation of differences in responses at two different points in the factor space is of great importance.

Park (1987) studied estimations of local slope (rate of change) of the response. He gave an example of the rate of change in the yield of the crop to various fertiliser doses. Das and Narasimham (1962) constructed many third-order rotatable designs by taking appropriate combinations of the symmetric point set or their suitably balanced subset obtained through balanced incomplete block designs and fractional replications. Huda (1987) constructed third-order rotatable designs in k-dimensions from those in lower dimensions. Koske et al. (2011) constructed a third-order rotatable design five-dimensions from a third-order rotatable design in lower dimensions through balanced incomplete block designs (BIBDs). Das et al. (1999) studied response surface design, symmetrical and asymmetrical rotatable and modified. Victorbabu (2011) explored a new method of construction of second order-slope-rotatable design using incomplete block designs with unequal block sizes. Victorbabu and Rajyalakshmi (2012) studied a new method of construction of robust second order rotatable designs using balanced incomplete block designs. Furthermore, Mutai Koske and Mutiso (2012) constructed some new four-dimensional third-order rotatable design through balanced incomplete design.

The method used by Huda (1987) Koske et al. (2011) and Mutai et al. (2012) can be used to obtain a generalised method of constructing designs.

METHODOLOGY

To construct a second-order rotatable design, a combination with unknown constants is taken and associated with 2^k factorial combinations of factors each at ± 1 levels to make level codes equidistant. All such combinations form a design. An incidences matrix of BIB is chosen suitable to satisfy the moment of rotatability i.e.

(i) $\sum_{u=1}^{N} x_{iu}^{2} = N\lambda_2 = A$ and

(ii) $\sum_{u=1}^{N} x_{iu}^{4} = 3\sum_{u=1}^{N} x_{iu}^{2} x_{ju}^{2} = 3N\lambda_4 = 3B$

Where all other sums of odd powers and cross products up to order four are zeros i.e.

$\sum_{u=1}^{N}\prod_{i=1}^{v} x_{iu}\alpha_i = 0$ if any α_i is odd for $\sum \alpha_i \leq 4$

These arrangements of points are said to form a rotatable design of second-order only if it forms a non-singular second-order design.

These points should give rise to a non-singular $X'X$ matrix.

Variances and covariances

The variances and covariances of parameter estimates are obtained by determining the inverse of the moment matrix.

Conditions for Rotatability

Variances and Covariances of the parameter estimates

The variances and covariances are obtained by determining the inverses of the diagonal sub-matrices of the moment matrix M.

$$M^{-1} = \begin{bmatrix} E^{-1} & 0 & 0 \\ & F^{-1} & 0 \\ Symmetric & & F^{-1} \end{bmatrix}$$

Consider the matrix $\underline{X} = \begin{bmatrix} \underline{x_0} \\ \underline{x_1} \\ \underline{x_2} \\ . \\ . \\ x_k \end{bmatrix}$ then M is obtained as $\frac{1}{N} X'X$

$$
\left[\begin{array}{cccccc:ccccc:ccccc}
x_0^2 & x_0^2x_1^2 & x_0^2x_2^2 & . & . & . \; x_0^2x_k^2 & & & & & & & & & & \\
 & x_1^4 & x_1^2x_2^2 & . & . & . \; x_1^2x_k^2 & & & & & & & & & & \\
 & & x_2^4 & . & . & . \; x_2^2x_k^2 & & & H & & & & & I & & \\
 & & & . & & . & & & & & & & & & & \\
 & & & & . & . & & & & & & & & & & \\
 & & & & & . \; . & & & & & & & & & & \\
 & & & & & x_k^2 & & & & & & & & & & \\
\hdashline
 & & & & & & x_1^2x_2^2 & x_1^2x_2^2x_3 & . & . & . \; x_1^2x_2^2x_3 & & & & & \\
 & & & & & & & x_2^2x_3^2 & . & . & . \; x_2^2x_k^2 & & & & & \\
 & & & & & & & & & & & & & J & & \\
\hdashline
 & & & & & & & & & & & x_1^2 & x_1x_2 & . & . & . \; x_1x_k \\
 & & & & & & & & & & & & x_2^2 & . & . & . \; x_2x_k \\
 & & Symmetric & & & & & & & & & & & . & & . \\
 & & & & & & & & & & & & & & . & . \\
 & & & & & & & & & & & & & & & . \; . \\
 & & & & & & & & & & & & & & & x_k^2
\end{array}\right]
$$

which simplifies to

$$
\left[\begin{array}{cccccccc:cccccc:cccccc}
1 & \lambda_2 & \lambda_2 & . & . & . & \lambda_2 & & & & & & & & & & & & \\
 & 3\lambda_2 & \lambda_4 & . & . & . & \lambda_4 & & & & & & & & & & & & \\
 & & 3\lambda_4 & . & . & . & \lambda_4 & & & & & & & & & & & & \\
 & & & . & & & . & & & & H & & & & & & I & & \\
 & & & & . & & . & & & & & & & & & & & & \\
 & & & & & . & . & & & & & & & & & & & & \\
 & & & & & & 3\lambda_4 & & & & & & & & & & & & \\
\hdashline
 & & & & & & & \lambda_4 & 0 & . & . & . & 0 & & & & & & \\
 & & & & & & & & \lambda_4 & . & . & . & 0 & & & & & & \\
 & & & & & & & & & . & & & . & & & & J & & \\
 & & & & & & & & & & . & & . & & & & & & \\
 & & & & & & & & & & & . & . & & & & & & \\
 & & & & & & & & & & & & \lambda_4 & & & & & & \\
\hdashline
 & & & & & & & & & & & & & \lambda_2 & & & & & \\
 & & & & & & & & & & & & & & \lambda_2 & & & & \\
 & & Symmetric & & & & & & & & & & & & & . & & & \\
 & & & & & & & & & & & & & & & & . & & \\
 & & & & & & & & & & & & & & & & & . & \\
 & & & & & & & & & & & & & & & & & & \lambda_2
\end{array}\right]_{(k+1)\times(k+1)}
$$

Where H, I and J are null matrices, and

$$
E=\begin{bmatrix}
1 & \lambda_2 & \lambda_2 & . & . & . & \lambda_2 \\
 & 3\lambda_4 & \lambda_4 & . & . & . & \lambda_4 \\
 & & 3\lambda_4 & . & . & . & \lambda_4 \\
 & & & . & & & . \\
 & & & & . & & . \\
 & & & & & . & . \\
 & & & & & & 3\lambda_4
\end{bmatrix}
$$

$$F = \begin{bmatrix} \lambda_4 & 0 & . & . & . & 0 \\ & \lambda_4 & . & . & . & 0 \\ & & . & & & . \\ & & & . & & . \\ & & & & . & . \\ & & & & & \lambda_4 \end{bmatrix} \text{ and}$$

$$G = \begin{bmatrix} \lambda_2 & 0 & . & . & . & 0 \\ & \lambda_2 & . & . & . & 0 \\ & & . & & & . \\ & & & . & & . \\ & & & & . & . \\ & & & & & \lambda_2 \end{bmatrix}$$

In order to get the inverse of M, the inverses of the sub-matrices E, F and G were calculated. Considering the sub-matrices E let inverse of the matrix be

$$E_{(k+1)(k+1)} = \begin{bmatrix} a & b & b & . & . & . & b \\ & c & d & . & . & . & d \\ & & & . & & & . \\ & & & & . & & . \\ & & & & & . & . \\ & & & & & & c \end{bmatrix} \qquad (12)$$

$$\begin{bmatrix} 1 & \lambda_2 & \lambda_2 & . & . & . & \lambda_2 \\ & 3\lambda_4 & \lambda_4 & . & . & . & \lambda_4 \\ & & 3\lambda_4 & . & . & . & \lambda_4 \\ & & & . & & & . \\ & & & & . & & . \\ & Symmetric & & & & . & . \\ & & & & & & 3\lambda_4 \end{bmatrix} \begin{bmatrix} a & b & b & . & . & . & b \\ & c & d & . & . & . & d \\ & & c & . & . & . & d \\ & & & . & & & . \\ & & & & . & & . \\ & Symmetric & & & & . & . \\ & & & & & & d \end{bmatrix} == I_{k+1} \qquad (13)$$

$$a + b\lambda_2 + b\lambda_2 + \cdots + b\lambda_2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$a\lambda_2 + 3b\lambda_4 + b\lambda_4 + \cdots + b\lambda_4 = 0 \qquad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

$$b\lambda_2 + 3c\lambda_4 + d\lambda_4 + \cdots + d\lambda_4 = 1 \qquad \ldots\ldots\ldots\ldots\ldots \text{(iii)} \qquad (14)$$

$$b\lambda_2 + c\lambda_2 + d\lambda_2 + \cdots d\lambda_2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

i.e.

$a + \ kb\lambda_2 \ = 1$ .... (i)

$a\lambda_2 + (k+2)b\lambda_4 = 0$ .... (ii)

$b\lambda_2 + 3c\lambda_4 + (k-1)d\lambda_4 = 1$ .... (iii) (15)

$b + c\lambda_2 + (k+2)d\lambda_4 = 0$ .... (iv)

Solving the equations gives

From Eqn 15(i) $a = 1 - kb\lambda_2,$

Substituting in equation 15(ii)

$$(1 - kb\lambda_2)\lambda_2 + (k+2)b\lambda_4 = 0$$

and simplifying we obtain

$$b = \frac{\lambda_2}{k\lambda_2{}^2 - (k+2)\lambda_4} \quad (16)$$

substituting the value of$b$, in equation 15(i), we obtain

$$a = \frac{-(k+2)\lambda_4}{k\lambda_2{}^2 - (k+2)\lambda_4} \quad (17)$$

From Eqn. (iii) and Eqn. (iv)

$$c = -\left(\frac{b + (k-1)d\lambda_2}{\lambda_2}\right)$$

and $$c = \frac{1 - b\lambda_2 - (k-1)d\lambda_4}{3\lambda_4}$$

solving the two equations above we get

$$d = \frac{b\lambda_2{}^2 - \lambda_2 - 3b\lambda_4}{2(k-1)\lambda_2\lambda_4}$$

and substituting for b

i.e. $$b = \frac{\lambda_2}{k\lambda_2{}^2 - (k+2)\lambda_4}$$

then, $$d = \frac{\left[\frac{\lambda_2}{k\lambda_2{}^2 - (k+2)\lambda_4}\right]\lambda_2{}^2 - \lambda_2 - 3\lambda_4\left[\frac{\lambda_2}{k\lambda_2{}^2 - (k+2)\lambda_4}\right]}{2(k-1)\lambda_2\lambda_4}$$

and solving the equation we get

$$d = \frac{(\lambda_4 - \lambda_2^{\;2}}{2\lambda_4\left[k\lambda_2^{\;2} - (k+2)\lambda_4\right]}$$

(18)

From

$$c = -\left(\frac{b + (k-1)d\lambda_2}{\lambda_2}\right)$$

and solving for c by substituting b and d

i.e. $$c = -\frac{\left\{\dfrac{\lambda_2}{k\lambda_2^{\;2} - (k+2)\lambda_4} + (k-1)\lambda_2\left[\dfrac{\left(\lambda_4 - \lambda_2^{\;2}\right)}{2\lambda_4\left[k\lambda_2^{\;2} - (k+2)\lambda_4\right]}\right]\right\}}{\lambda_2}$$

solving the equation above, the value of c is given by

$$c = \frac{(k-1)\lambda_2^{\;2} - (k+1)\lambda_4}{2\lambda_4\left[k\lambda_2^{\;2} - (k+2)\lambda_4\right]} \qquad (19)$$

The values of a, b, c and d obtained in equation 16 to 19 will give the inverse of E as

$$E^{-1} = \frac{1}{[2\lambda_4][k\lambda_2^{\;2} - (k+2)\lambda_4]}\begin{bmatrix} -(k+2)\lambda_4^{\;2} & 2\lambda_2\lambda_4 & 2\lambda_2\lambda_4 & . & . & 2\lambda_2\lambda_4 \\ & (k-1)\lambda_2^{\;2} - (k+1)\lambda_4 & \lambda_4 - \lambda_2^{\;2} & . & . & \lambda_4 - \lambda_2^{\;2} \\ & & (k-1)\lambda_2^{\;2} - (k+1)\lambda_4 & . & . & \lambda_4 - \lambda_2^{\;2} \\ & & & . & . & . \\ & Symmetric & & . & . & . \\ & & & & & (k-1)\lambda_2^{\;2} - (k+1)\lambda_4 \end{bmatrix}$$

$$(k+1)\times(k+1)$$

(20)

considering the submatrix $$F = \begin{bmatrix} \lambda_4 & 0 & 0 & . & . & . & 0 \\ & \lambda_4 & 0 & . & . & . & 0 \\ & & \lambda_4 & . & . & . & 0 \\ & & & . & & & . \\ & & & & . & & . \\ & Symmetric & & & & . & . \\ & & & & & & \lambda_4 \end{bmatrix}$$

then the inverse of F is given as

$$F^{-1} = \begin{bmatrix} \lambda_4^{-1} & 0 & 0 & . & . & . & 0 \\ & \lambda_4^{-1} & 0 & . & . & . & 0 \\ & & \lambda_4^{-1} & . & . & . & 0 \\ & & & . & & & . \\ & & & & . & & . \\ & Symmetric & & & & . & . \\ & & & & & & \lambda_4^{-1} \end{bmatrix} \qquad (21)$$

and

$$G = \begin{bmatrix} \lambda_2 & 0 & 0 & . & . & . & 0 \\ & \lambda_2 & 0 & . & . & . & 0 \\ & & \lambda_2 & . & . & . & 0 \\ & & & . & & & . \\ & & & & . & & . \\ & Symmetric & & & & . & . \\ & & & & & & \lambda_2 \end{bmatrix}$$

the inverse of G is given as

$$G^{-1} = \begin{bmatrix} \lambda_2^{-1} & 0 & 0 & . & . & . & 0 \\ & \lambda_2^{-1} & 0 & . & . & . & 0 \\ & & \lambda_2^{-1} & . & . & . & 0 \\ & & & . & & & . \\ & & & & . & & . \\ & Symmetric & & & & . & . \\ & & & & & & \lambda_2^{-1} \end{bmatrix} \qquad (22)$$

therefore

$$M^{-1} = \begin{bmatrix} D^{-1} & 0 & 0 \\ & E^{-1} & 0 \\ Symmetric & & F^{-1} \end{bmatrix} \qquad (23)$$

The variances and covariances of the second-order polynomial were obtained by utilising the inverse of the moment matrix already obtained in equation (23).

Using the least-squares estimation equation$X'X\underline{\beta} = X'\underline{Y}$, then if both sides are premultiplied by $(X'X)^{-1}$ we obtain

$$(X'X)^{-1}X'X\underline{\beta} = (X'X)^{-1}X'\underline{Y}$$

$$\therefore \ \beta = (X'X)^{-1}X'\underline{Y}$$

and the estimated variance is given by

$$var\left(\hat{\beta}\right) = (X'X)^{-1}(X'X)(X'X)^{-1}\sigma^2 I_{k+1} = (X'X)^{-1}\sigma^2 I_{k+1}$$

$$= M^{-1}\sigma^2$$

and from M^{-1} that was obtained from equation (23) the variances and covariances of the estimated response parameters are given by

$$\mathrm{var}(b_0) = \frac{-(k+2)\lambda_4{}^2}{2\lambda_4[k\lambda_2{}^2 - (k+2)\lambda_4]}\sigma^2$$

$$\mathrm{var}(b_i) = \frac{1}{\lambda_2}\sigma^2$$

$$\mathrm{var}(b_{ij}) = \frac{1}{\lambda_4}\sigma^2$$

$$\mathrm{var}(b_{ii}) = \frac{(k-1)\lambda_2{}^2 - (k+1)\lambda_4}{2\lambda_4[k\lambda_2{}^2 - (k+2)\lambda_4]}\sigma^2$$

$$\mathrm{cov}(b_0, b_{ii}) = \frac{\lambda_2\lambda_4}{\lambda_4[k\lambda_2{}^2 - (k+2)\lambda_4]}\sigma^2$$

$$\mathrm{cov}(b_{ii}, b_{jj}) = \frac{\lambda_4 - \lambda_2{}^2}{2\lambda_4[k\lambda_2{}^2 - (k+2)\lambda_4]}\sigma^2 \qquad (24)$$

The variances and covariances for the modified conditions $\lambda_2{}^2 = \lambda_4$ give

$$\mathrm{var}(b_0) = \frac{-(k+2)}{4}\sigma^2$$

$$\mathrm{var}(b_i) = \frac{1}{\lambda_2}\sigma^2$$

$$\mathrm{var}(b_{ij}) = \frac{1}{\lambda_4}\sigma^2$$

$$\mathrm{var}(b_{ii}) = \frac{1}{2\lambda_4}\sigma^2$$

$$\mathrm{cov}(b_0, b_{ii}) = \frac{-1}{2\lambda_2}\sigma^2$$

$$\mathrm{cov}(b_{ii}, b_{jj}) = 0 \qquad (25)$$

RESULTS

The variances and covariances of the parameter estimates of a second-order experimental design were also obtained in equation (24) and under the modified conditions $\lambda_4 = {\lambda_2}^2$in equation (25).

CONCLUSION

It is seen that if $\lambda_4 = {\lambda_2}^2$the variances and covariances of the parameter estimates become similar except the $var\ (b_0)$and the $cov\ (b_{ii}, b_{ij})$= 0. Therefore, the modification of the variances and covariances affect the estimated response. If an experimenter might be interested in some k subset of factors, then the results in these studies will be desirable since this subset will be identified with blocks generating a BIB (k, b, s, r, λ) so that the second-order k-dimensional design contains second order rotatable designs in k-1 dimensions involving the subsets of factors the experimenter is interested in. Where the experimenter is interested in replicate, then the design is suitable since a replicate of the incidence matrix of BIB is used. The experimental designs that were obtained in this study ensure equal precision on the response to cut down on cost. The design can be useful in agriculture, textile industry, motor vehicle industry and all other types of industry that make use of experimental designs to manufacture their products.

REFERENCES

Box, GEP, & Hunter, J.S (1957). Multi factor experimental designs for exploring response surfaces. *Annals of Mathematical Statistics, 28,* 195-241.

Das, M. N., & Narasimham, V.L (1962). Construction of rotatable designs through balanced incomplete block designs. *Annals of Mathematical Statistics, 33,* 1421-1439.

Huda, S. (1987). The construction of third-order rotatable designs in k dimension from those in the lower dimension. *Pak. J. Statistics, 3*(1), 11-16.

Koske, J. K., Mutiso, J. M., & Kosgei, M. K. (2011). New third-order rotatable design in five dimensions through balanced incomplete block design. *Journal of Agriculture Science and Technology, 13,* 157-161.

Mutai, C. K., Koske, J. K., & Mutiso, J. M (2012). Construction of a four-dimensional third-order rotatable design through balance incomplete block design. *Advances and Application in Statistics 27*(1),47-53

Park, S. H. (1987). A class of multifactor designs for estimating the slope of response surfaces. *Technometrics, 29,* 449-453.

Victorbabu, B. Re (2005). Modified slope rotatable central composite designs. *Journal of Korean Statistical Society, 34*:153-160.

Victorbabu, B. Re., & Narasimham, V.L (1991). Construction of second-order slope rotatable designs. *Communication in Statistics: Theory and Methods, 20,* 2467-2478.

Victorbabu, B. Re. (2006). Modified second order rotatable and slope rotatable design using a pair of incomplete block design. *Sri Lankan Journal of Applied Statistics, 7,* 39-53.

Victorbabu, B. Re. (2009). On modified second-order slope rotatable design. *Department of Statistics, Acharya Nagarjuna University, 2,* 115-131.

Victorbabu, B.R., & Rajyalakshmi, K. (2012). A New Method of Construction of Robust Second Order Slope Rotatable Designs Using Pairwise Balanced Designs.

Victorbabu, B. Re., & Surekha, Ch. VVS (2011). A new method of construction of second-order slope- rotatable designs using balanced incomplete block designs. *Department of Statistics, Acharya Nagarjuna University, 27*(3), 221-230.

Victorbabu, B. Re. and Vasundharadevi, V. (2009). On the efficiencies of second-order response surface designs for the estimation of responses and slopes using symmetrical unequal block arrangements with two unequal block sizes. *Japan Statistics Society, 39,* 1-14.

Victorbabu, Re. B (2011). A new method of construction of second-order slope rotatable designs using incomplete block design with unequal block sizes. *Probstat Forum 4,* 44-53.